19.02.2014 B.Sc. Part II (Hons)

4th paper

v.v. Impt & Difficult

Linear differential Equations with constant coefficients

Q: Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2 e^{2x} \sin 2x$

Soln For CF,

$$\frac{d^2}{dx^2} - 4\frac{d}{dx} + 4 = 0$$

$$\Rightarrow (D-2)^2 = 0 \Rightarrow D = 2, 2$$

$$\therefore CF = (C_1 + C_2 x) e^{2x} \quad \text{—— (1)}$$

Now we find P.I.

$$\therefore PI = \frac{1}{D^2 - 4D + 4} 3x^2 e^{2x} \sin 2x$$

First we integrate e^{2x}.

~~for this,~~ ~~put~~ replace D by D+a

i.e D+2.

$$\therefore PI = 3e^{2x} \frac{1}{[(D+2)-2]^2} x^2 \sin 2x$$

$$= 3e^{2x} \frac{1}{D^2} x^2 \sin 2x$$

$$= \text{Imaginary part of } 3e^{2x} \frac{1}{D^2} x^2 (\cos 2x + i \sin 2x)$$

$\therefore$ P.I. = I.P. of $3e^{2x} \frac{1}{D^2} e^{2ix} \cdot x^2$

Now integrate e^{2ix}. For this replace D by D+a ie. D+2i (here $a = 2i$).

$$\therefore PI = I.P. \text{ of } 3e^{2x} \cdot e^{2ix} \frac{1}{(D+2i)^2} x^2$$

$$\Rightarrow PI = I.P \text{ of } 3e^{2x} \cdot e^{2ix} \frac{1}{(D^2+4Di-4)} x^2$$

$$\Rightarrow PI = I.P. \text{ of } -\frac{3}{4} e^{2x} \cdot e^{2ix} \frac{1}{\left[1 - \frac{(D^2+4Di)}{4}\right]} x^2$$

$$\Rightarrow PI = I.P. \text{ of } -\frac{3}{4} e^{2x} \cdot e^{2ix} \left[1 - \frac{(D^2+4Di)}{4}\right]^{-1} x^2$$

$$\Rightarrow PI = I.P. \text{ of } -\frac{3}{4} e^{2x} \cdot e^{2ix} \left[1 + (4Di + D^2)/4 + \frac{(D^2+4Di)^2}{16} + \cdots\right] x^2$$

$$\Rightarrow PI = I.P. \text{ of } -\frac{3}{4} e^{2x} \cdot e^{2ix} \left[x^2 + (4Di + D^2) x^2/4 + \frac{(16D^2i^2) x^2}{16} + 0\right]$$

$\therefore$ PI = I.P. of $-\frac{3}{4} e^{2x} \cdot e^{2ix}\left[x^2 + (8ix+2)/4 - \frac{\cancel{16} \times 2}{\cancel{16}}\right]$

$\Rightarrow$ PI = I.P. of $-\frac{3}{4} e^{2x} \cdot e^{2ix}\left[x^2 + \frac{4ix+1}{2} - 2\right]$

$= \text{I.P of } -\frac{3}{4} e^{2x} \cdot e^{2ix}\left[x^2 + 2ix + \frac{1}{2} - 2\right]$

$= \text{I.P. of } -\frac{3}{4} e^{2x} \cdot (\cos 2x + i \sin 2x)\left(x^2 + 2ix - \frac{3}{2}\right)$

$= -\frac{3}{4} e^{2x}\left[2x \cos 2x + x^2 \sin 2x - \frac{3}{2} \sin 2x\right]$

$= -\frac{3}{8} e^{2x}\left[4x \cos 2x + 2x^2 \sin 2x - 3 \sin 2x\right]$

$\Rightarrow$ PI $= -\frac{3}{8} e^{2x}\left[(2x^2 - 3)\sin 2x + 4x \cos 2x\right]$ — (2)

Hence complete soln is given by

$y = CF + PI$

Where CF is given by (1) and PI by (2).